MAGNIFICATION BIAS IN GALACTIC MICROLENSING SEARCHES

Robert J. Nemiroff

George Mason University, CSI, Fairfax, VA 22030
NASA / Goddard Space Flight Center, Code 668.1, Greenbelt, MD 20771

In Press:
The Astrophysical Journal

ABSTRACT

It is shown that a significant amount of detectable gravitational microlensing events that could potentially be found by MAssively Parallel Photometry (MAPP) projects (such as the MACHO, EROS and OGLE collaborations) will occur for stars too dim to be easily noticed individually by these projects. This is the result of a large magnification bias effect, a bias of including high magnification events in any flux-limited sample. The probability of detecting these events may be as high as 2.3 times the lensing probability of stars currently being monitored by MAPP collaborations.

Subject Headings: stars: low-mass, brown dwarfs, Galaxy: halo, dark matter, gravitational lensing

1. Introduction

Recently Alcock et al. (1993, the MACHO collaboration), Aubourg et al. (1993a, the Experience de Recherche d'Objets Sombres or EROS project) and Udalski et al. (1993a, called Optical Gravitational Lensing Experiment or OGLE) have all reported seeing light curves of stars indicative of a fainter star passing in front of - and hence gravitationally magnifying - the light from a background star: a gravitational microlens event. The probability of seeing such an event was predicted originally by Paczynski (1986), Griest (1991), and Nemiroff (1991) based on the probability formalism developed in Nemiroff (1989). Assuming that the nature of these events is correctly identified, these events are measuring the mass and density of stars in our disk and dark-matter halo objects in the Galactic halo.

MAssively Parallel Photometry (hereafter referred to as MAPP) projects currently monitor known stars for variations indicative of gravitational lensing. The method currently used by the MACHO collaboration, as described in Alcock et al. (1992), the EROS collaboration, as described in Aubourg et al. (1993b), and the OGLE project, as described in Udalski et al. (1993b), appears to disregard lensing of stars that are too dim to be easily noticed in the absence of a magnification event.

The general luminosity function of stars slightly dimmer than those being monitored by current MAPP projects clearly shows that there are increasingly more stars at dimmer magnitudes. Although more numerous, these dim stars must undergo a stronger lens effect to be even potentially noticeable by a MAPP search. The first group to consider the effects of these dim stars was Bouquet, Kaplan, Melchior, Giraud-Heraud, and Baillon (1994). They presented numerical Monte-Carlo results showing that monitoring pixels instead of stars could provide a ten-fold increase in detection efficiency, particularly in the EROS CCD program. It will be shown here by simple analytical estimates that the probability of detection of lensing of these dim stars is indeed substantial for all of the

MAPP projects, although the probability is a strong function of the luminosity function of the source stars involved, which has significant uncertainty.

A possible future MAPP mission of M31 has been suggested by Crotts (1992). Crotts' suggested procedure for finding M31 microlensing is similar to the proposed method here in that it depends on the monitoring of a whole stellar-rich field rather than monitoring a previously determined list of stellar lensing candidates. In this work we apply this "field monitoring" concept to the present MAPP missions monitoring the LMC and the galactic bulge. We show explicitly that the lens detection rate is a function of the monitoring rate and the luminosity function of stars slightly below the detection threshold in the source field. This this type of lensing phenomenon is well known in gravitational lensing as "magnification bias." We estimate the potential increase in lens detections of the currently running MAPP missions.

Section 2 will discuss generally the probability of these dim stars being detectably gravitationally lensed. Section 3 will apply this analysis to currently operating MAPP projects. Section 4 will provide discussion.

2. Magnification Bias and MAPP Searches

Magnification bias occurs for a flux-limited sample by the natural inclusion of sources normally too faint to make the sample but included fortuitously because of being magnified by a foreground gravitational lens (Blandford and Narayan 1992; Narayan 1994). This bias is particularly important for sources with a luminosity function which shows a great increase in the number of sources marginally fainter than the flux-limit.

Nemiroff (1991) showed that for sources monitored on a time-scale short compared to the duration of a lens event, the waiting time between lensing events of magnification factor A is, when A is large,

$$t_{wait} \propto \frac{A}{N}, \tag{1}$$

where N is the number of stars being monitored. This waiting time depends on the relative motion of the lens and the sources and therefore its dependence on A is qualitatively different than discussed in the static cases of Paczynski (1986), Griest (1991) and Nemiroff (1991), although they note that the classically defined optical depth to gravitational lensing is small allowing single lens models. Note also that although the probability of lensing above a given magnification is proportional to optical depth, optical depth itself does not include relative lens and source motion and is tied to a specific magnification. Therefore optical depth will not be explicitly considered here. Because of this, I will digress and discuss a simple method to see Eq. 1 intuitively.

Consider a simple adaptation of classical mean free path arguments to gravitational lensing probabilities. Around each lens, picture an angular circle within which a background star must pass to undergo a detectable magnification. Now consider the relative angular motion between lenses and background stars as potentially visible to the observer. A background star will experience this relative angular motion and therefore has a two dimensional "mean free path" l before encountering a circle around a lens which will magnify it detectably. In general, for a single source, $l \propto (\sigma B)^{-1}$ where σ is the (unknown) angular two-dimensional number density of lenses, each with 'lens detection' cross-section B. For large magnification events, $B \propto A^{-1}$, so that $l \propto A$. For a canonical relative angular velocity between lenses and sources, the mean time between events $t_{wait} \propto l$ so that $t_{wait} \propto A$. Clearly the waiting time between lensing events for N sources scales as $1/N$, so that the form or Eq. (1) is verified. The probability that a source in an ensemble of N sources will undergo detectable lensing is directly proportional to the size of the ensemble and inversely proportional to the waiting time, so that $P \propto N/A$.

We now assume that there is a strict limiting magnitude for source stars above which a MAPP search will surely monitor a star and below which a MAPP search will definitely not monitor a star. To simplify terminology, we will adopt the term "bright" to refer to stars brighter this limiting magnitude, and the term "dim" to refer to stars dimmer than this magnitude.

For a dim star to be detectably lensed, we will assume that the star must be magnified by a factor A_{dim} to reach the magnitude limit that divides the bright and dim groups, and an additional magnification factor A_{MAPP} for lensing to be strong enough to be detectable. One might immediately think that lensing of the dim group is quite unlikely, relative to the bright group, but this may be offset by the greater number of stars in the dim group.

From the above assumptions, a dim star originally a factor A_{dim} below a MAPP detection limit could potentially be seen to undergo detectable lens magnification if it is magnified by at least the amount $(A_{MAPP} + A_{dim})$. For stars well below the detection limit A_{dim} will be considered much greater than A_{MAPP}, so that $P_{dim} \propto N_{dim}/A_{dim}$. Now A_{dim} is equivalent to the brightness of the detection limit divided by the brightness of the dim star: L_{lim}/L_{dim}, where L_{lim} is the limiting absolute luminosity of the MAPP region. Assuming that the luminosity function can be characterized as $N \propto L^{\alpha}$ so that $N_{dim}/N_{lim} = (L_{dim}/L_{lim})^{\alpha}$ then

$$\frac{P_{dim}}{P_{lim}} \propto \frac{(N_{dim}/N_{lim})}{A_{dim}} \propto L_{dim}^{\alpha+1}. \tag{2}$$

If the source stellar luminosity function has $\alpha < -1$, the probability of dim star lensing actually *increases* as the apparent brightness of the stars decrease. This shows that magnification bias is not only important - it may help create a more likely form of lens detection.

If Eq. (2) continued to even dimmer stars indefinitely, dim star lensing would be very much *more* probable than detection of lensing from the bright stars. In this case, the dimmer the star, the more of them there are, the higher the magnification needed for detection, but the more detections per time there should be. Practically, this should continue until the luminosity function of stars in the MAPP project becomes flatter than

L^{-1}, at which point the decrease in the probability of high magnification microlensing becomes more important than the increase in the number of stars at fainter magnitudes.

3. Application to Existing MAPP Projects

Alcock et al. (1992) give a limiting magnitude for the MACHO collaborations photometry of about 19.5 for a 5 minute exposure (at full moon). Aubourg et al. (1993b) estimate a mean visual magnitude for their photographic search of 19.0. Udalski et al. (1993b) estimate a limiting visual magnitude for the OGLE project of about 19.5. We assume a distance modulus in the V band to the LMC of $V = 18.5$ (Blaha and Humphreys 1989). Therefore the absolute visual magnitude of an LMC star at the limiting magnitude of the MACHO and EROS collaborations is about $M_V = 1$.

Unfortunately, the intrinsic luminosity function of stars of this apparent magnitude and fainter has significant uncertainties. We first consider the luminosity function of the LMC stars. Table 4-5 in Mihalas and Binney (1981), drawn from results obtained by Luyten (1968), gives the general luminosity function of stars in the Galactic disk. We will follow Blaha and Humphreys (1989), who showed that the LMC and the Galactic disk have similar luminosity functions for O and B stars in the range $-9 < M_V < -7$, and assume that this similarity applies to dimmer stars as well.

We use Eq. (2) to estimate the probability of dim star microlensing by the luminosity function until $M_V = 10$, which is well past the magnitude where additional lensing probability is substantial. This calculation shows that there should be about 2.3 times the number of lens events (of any magnification) involving stars too dim to be continually monitored than involving stars currently monitored by the MACHO or EROS collaborations.

We note that the Luyten luminosity function's slope rises above -1 coincidently at $M_V = 1$, the magnitude limit of the MACHO collaboration search. Were the MACHO and EROS searches not as deep, the magnitude of the magnification bias would be expected to be much stronger. As it is, were EROS's magnitude limit fully 1/2 magnitude brighter than MACHO's, EROS would be susceptible to about an additional 25 % of lensing events occurring on stars down to only 1/2 magnitude below this limit.

Things do not look as promising for the OGLE project in this respect. The luminosity function of stars in Baade's window, as measured by Holtzman et al. (1993), again coincidently shows a break near the completeness limit, but this time at a slightly dimmer magnitude of about $M_V = 20.0$. Stars dimmer than this break have a luminosity function too flat for them to be significant lens candidates. Between the OGLE magnitude limit of 19.5 and this break, however, there might be a significant magnification bias effect. From Figure 5 of Holtzman et al. (1993), we estimate a luminosity function with slope of about -1.25 in the range $19.5 < M_V < 20.0$, although the plotted error bars show significant uncertainty in this slope. From Eq. (2) we would estimate that there should be about an equal number of detectable lens events (of any magnification) involving stars too dim to be continually monitored than involving stars currently monitored by the OGLE collaboration.

4. Discussion

Lensing of stars below the detection threshold could be found in anomalous brightening events not matched with known stars recorded by the MACHO, EROS and OGLE collaborations. The source stars involved with these events might be recovered by a large telescope after the lensing event. Current detection techniques necessarily select large magnification events for these dim stars. Since only during times of high magnification would these events be visible, the duration of this type of lensing event would be less than the "normal" MACHO event. This reduced duration might result in a potential "duration" bias against the detection of these events.

It is interesting to note that the *static* probability of lensing detection - that not assuming relative lens and source motion - would predict that magnification bias is unimportant. This is because the static probability of lensing magnification is proportional to A^{-2} (at large A), so that it is much less probable for high lens magnification that the increase in the number of potential sources given by the stellar luminosity function of the LMC would not compensate. Only when considering the *dynamic* probability - which includes relative motion between the source and lens populations in the probability estimate, does it become clear that magnification bias may create a sizeable effect. For the dynamic lens paradigm to be in effect, however, the time between measurements of apparent stellar brightness must be small compared to the duration of the lensing event.

Note that the probabilities estimated here assume that the source star had to be magnified above the MAPP projects' current magnitude limit to be considered a microlensing candidate. Were a MAPP project begun using a telescope with higher angular resolution, the magnitude limit would be extended to dimmer stars, and each dim star need only undergo a magnification of A_{MAPP} to be considered a microlensing candidate. Then the probability of detectable lensing would clearly rise much above that reported here - it would then be just roughly proportional to the increased number of stars being monitored.

Lensing of stars below a limiting magnitude works best when this magnitude occurs at relatively bright intrinsic stellar luminosities. At these luminosities the number-magnitude relation is very steep - describing many more stars in successively fainter magnitude intervals. In this vein, a proposed project for monitoring stars in M31 (Crotts 1994) would be quite susceptible to this type of magnification bias effect.

Magnification bias comes about because of a magnitude limit, a magnitude limit is usually used in MAPP searches because of source confusion, source confusion results from the angular resolution limit not being able to resolve stars from each other, and angular resolution limits at this level result from the Earth's atmosphere. Therefore were a space-based MAPP project implemented, one might be able to extend the magnitude limit down to a stellar brightness where the luminosity function would rise slow enough such that magnification bias would not be important. The main value of a space-based MAPP project would be, of course, that many more stars could be continually monitored for gravitational microlensing. Failing this, magnification bias will continue to be an important effect.

In sum, currently operating searches for gravitational microlensing by stars in our Galaxy suffer a large magnification bias. This bias translates into a significant probability that stars too dim to be included in an individual monitoring program would be magnified above detection limits. Existing and future MAPP type searches might consider adapting their data acquisition techniques and software to include the possibility of detecting such events.

This work was supported by a grant from NASA.

REFERENCES

Alcock, C. A. et al. 1992, Astron. Soc. Pac. Conf. Ser., 34, 193

Alcock, C. A. et al. 1993, Nature, 365, 621

Aubourg, E. et al. 1993a, Nature, 365, 623

Aubourg, E. et al. 1993b, Messenger, 72, 20

Blaha, C. and Humphreys, R. A. 1989, AJ 98, 1598

Blandford, R. and Narayan, R., 1992, Ann. Rev. Astron. Astrophys. 30, 311

Bouquet, A., Kaplan, J., Melchior, A.-L., Giraud-Heraud, Y., and Baillon, P. 1994, in proceedings of "The Dark Side of the Universe . . .", in press

Crotts, A. P. S. 1992, ApJ, 399, L43

Crotts, A. P. S. 1994, in Proceedings of the 31st International Liege Colloquium: Gravitational Lensing in the Universe, in press.

Griest, K. 1991, ApJ, 366, 412

Holtzman, J. A. et al. 1993, AJ, 106, 1826

Luyten, W. J. 1968, MNRAS, 139, 221

Mihalas, D. and Binney, J. 1981, Galactic Astronomy, Structure and Kinematics, 2nd

Edition, (San Francisco, Freeman), Chapter 4

Narayan, R. 1994, in Proceedings of the 31st International Liege Colloquium: Gravitational Lensing in the Universe, in press.

Nemiroff, R. J. 1989, ApJ, 341, 579

Nemiroff, R. J. 1991, A&A, 247, 73

Paczynski, B. 1986, ApJ, 304, 1

Udalski, A., Szymanski, M., Kaluzny, J., Kubiak, M., Krzeminski, W., Mateo, M., Preston, G. W., and Paczynski, B. 1993a, Acta Astron., 43, 289

Udalski, A. Szymanski, M., Kaluzny, J., Kubiak, M., and Mateo, M. 1993b, Acta Astron., 43, 69